Abstracts

Spinor field equations — A variational approach

Tian Xu
Tianjin University

We will discuss the existence of solutions for nonlinear problems arising from spin geometry. The problems in question are first order differential equations involving the Dirac operator. The nonlinearity will contain critical Sobolev exponent and is of particular geometric importance in dimension two. The picture of the existence of solutions for such problems is, except for some special cases, not very clear. In this talk, we will present some recent result in this direction

Perturbed Dirac-harmonic maps into flat tori: existence and multiplicity

Takeshi Isobe
Hitotsubashi University

Dirac-harmonic map is a mathematical model of supersymmetric σ-model arising in quantum filed theory. I will talk about some existence and multiplicity of perturbed Dirac-harmonic maps base on mini-max methods in critical point theory when the target is flat tori. If time permits, I will also discuss another approach based on Morse homology.

Blow up of solutions for a fourth order parabolic equation with gradient nonlinearlity

Nobuhito Miyake
Tohoku University

We consider the Cauchy problem for a fourth order nonlinear parabolic equation

$$\partial_t u + (-\Delta)^2 u = -\nabla \cdot (|\nabla u|^{p-2} \nabla u), \quad x \in \mathbf{R}^N,\ t > 0,$$

where $N \geq 1$ and $p > 2$. The aim of this talk is to show the existence of the solution u whose maximal existence time T_M is finite and to study the behavior of $u(t)$ and $\nabla u(t)$ at $t = T_M$. We also mention that gradient blow up phenomenon may occur, i.e., $\nabla u(t)$ blows up at $t = T_M$ and while $u(t)$ stay bounded at $t = T_M$. This talk is based on a joint work with Professor K. Ishige of the University of Tokyo and Professor S. Okabe of Tohoku University.

Behavior of the torsion function on a large spherical cap

Yoshitsugu Kabeya
Osaka Prefecture University

Let $N(\geq 2)$ be an integer and $\epsilon > 0$ is sufficiently small. In this talk, we consider properties of solutions to

$$\begin{cases} -\Lambda u &= 1, \quad \text{in } \Omega_\epsilon, \\ u &= 0, \quad \text{on } \partial\Omega_\epsilon, \end{cases} \tag{1}$$

where Ω_ϵ is a spherical cap on the unit sphere $\mathbb{S}^N(\subset \mathbb{R}^{N+1})$ whose center is at the North pole and its radius $(1-\epsilon)\pi$ and Λ is the Laplace-Beltrami operator on the unit sphere. A unique solution to (1) is called a "torsion function".

We expand the torsion function to by using the eigenfunctions of $-\Lambda$ under the homogeneous Dirichlet condition. We show how the solution behaves as $\epsilon \to +0$.

This talk is based on the joint project with Professor V. Moroz at Swansea University, UK.

On the traveling waves for surface diffusion of curves with constant contact angles

Yoshihito Kohsaka
Kobe University

The traveling waves for surface diffusion of plane curves are studied in this talk. The surface diffusion equation is given by

$$V = -\kappa_{ss},$$

where V and κ are the normal velocity and the curvature of an evolving plan curve, respectively, and s is the arc-length parameter. This geometric evolution equation can be represented as the 4th order nonlinear parabolic PDE. We consider an evolving plane curve with two endpoints which can move freely on the x-axis with generating constant contact angles. For the evolution of this plane curve governed by surface diffusion equation, the existence, the uniqueness and the convexity of traveling waves will be discussed.

Local existence and nonexistence for reaction-diffusion systems with coupled exponential nonlinearities

Masamitsu Suzuki
The University of Tokyo

We study the reaction-diffusion system with coupled exponential nonlinearities

$$\begin{cases} \partial_t u = \Delta u + e^{p_1 u + p_2 v} & \text{in } \mathbb{R}^N \times (0,T), \\ \partial_t v = \Delta v + e^{q_1 u + q_2 v} & \text{in } \mathbb{R}^N \times (0,T), \\ u(x,0) = u_0(x), v(x,0) = v_0(x) & \text{in } \mathbb{R}^N, \end{cases}$$

where $T > 0$, $p_i \geq 0$ and $q_i \geq 0$ $(i = 1, 2)$ with $(p_1, p_2) \neq (0, 0)$ and $(q_1, q_2) \neq (0, 0)$. The initial functions u_0 and v_0 are nonnegative and measurable. For each (p_1, p_2, q_1, q_2), we obtain integrability conditions of (u_0, v_0) which explicitly determine the existence/nonexistence of a local in time nonnegative classical solution. Our analysis can be applied to other nonlinearities including superexponential ones.

Existence and non-existence of the asymmetrical rotating solution of the reaction-diffusion particle model

Mamoru Okamoto
Hokkaido University

A particle-reaction-diffusion equation, which is a complex of ODE and PDE, is studied as a model of a self-propelled objects. The preceding study shows the equation has an non-trivial traveling wave solution numerically and experimentally, but the solution seems to be against the known mechanism of such a self-propulsion. We show the sufficient condition for existence and non-existence of the solution, and clarify the gap of the equation and the mechanism.

Gradient estimates for the heat equation with inverse-square potential

Yujiro Tateishi
The University of Tokyo

In this talk I will discuss gradient estimates for the solution to the initial value problem

$$\begin{cases} \partial_t u - \Delta u + V(|x|)u = 0 & \text{in} \quad \mathbf{R}^N \times (0, \infty), \\ u(x,0) = u_0(x) & \text{on} \quad \mathbf{R}^N, \end{cases}$$

where V is an inverse square radially symmetric potential and the operator $H := -\Delta + V$ is nonnegative on $L^2(\mathbf{R}^N)$. The inverse square potential is a typical one appearing in the study of the Schrödinger operators and it often appears in the linearized analysis for nonlinear diffusion equations and in the asymptotic analysis for diffusion equations.

The study of the decay estimates for the solutions to parabolic equations is classical subject. However it is difficult problem especially in the case that equation has strongly singular coefficients. The aim of this talk is to clarify a mechanism of the determination of the decay rate of ∇e^{-tH} by using the behavior of the positive harmonic function for the operator H.

Type II blowup solutions for the semilinear heat equation in 5D and 6D

Junichi Harada
Akita University

We discuss the existence of type II blowup solutions to the 5D and 6D semilinear heat equation.

$$u_t = \Delta u + |u|^{\frac{4}{n-2}} u \qquad \text{in } \mathbb{R}^n \times (0, T).$$

This result gives an analytical proof for the formal computation given by Filippas-Herrero-Velázquez (2000).

Asymptotic stability of stationary solutions to the drift-diffusion model with the fractional dissipation

YUUSUKE SUGIYAMA
The University of Shiga Prefecture

In this talk, we consider the following Cauchy problem for the drift-diffusion equation arising from a model of semiconductors:

$$\begin{cases} \partial_t u + (-\Delta)^{\theta/2} u - \nabla \cdot (u \nabla \psi) = 0, \\ -\Delta \psi = u - g, \\ u(0,x) = u_0(x), \end{cases}$$

First we prove the existence and uniqueness of the small solution to the corresponding stationary problem in the whole space. Next we consider the stability of stationary solutions. Namely it is proved that the unique solution of non-stationary problem exists globally in time and decays exponentially, if initial data is suitably close to the stationary solution and the stationary solution is sufficiently small. Furthermore, we will consider the drift-diffusion system for a model of bipolar semiconductor devices. We prove the stability of stationary solutions with a polynomial decay.

On the heat equation with a dynamic Hardy-type term

Eiji Yanagida
Tokyo Institute of Technology

Motivated by the work of Baras and Goldstein [1], we discuss the existence and non-existence of singular solutions for the heat equation with a time-dependent Hardy-type term, particularly, when the singular point is moving. It is shown that there exist two types of solutions in the subcritical case, and there are no positive solutions in the supercritical case. It turns out that various critical values play an essential role.

This talk is based on a joint work with Jann-Long Chern, Gyeongha Hwang and Jin Takahashi.

[1] P. Baras and J. A. Goldstein, *The heat equation with a singular potential*, Trans. Amer. math. Soc. 284 (1984), 121–139.