One-Day Workshop on Nonlinear PDEs at Ryukoku

Period: June 11 (Thursday) 13:10 – 19:00

Place: Lecture Room 1-614, Seta Campus, Ryukoku University

—Program—

13:00 – 13:10 opening

13:10 – 14:00 Yifu Zhou (Wuhan University) (*online)
Global solutions to the 4D Yang-Mills heat flow

14:20 – 15:10 Erbol Zhanpeisov (Tohoku University)
Blow-up rate for the subcritical semilinear heat equation in non-convex domains

15:30 – 16:20 Junichi Harada (Akita University)
Asymptotic behavior of solutions to the Fujita equation in 5D and 6D

16:40 – 17:30 Yannick Sire (Johns Hopkins University)
Harmonic maps into singular spaces

17:30 – 17:40 closing

18:00 – 19:00 discussion

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Organizers:
Tatsuki Kawakami (Ryukoku University)
Yohei Fujishima (Shizuoka)

Abstract

Global solutions to the 4D Yang-Mills heat flow

Yifu Zhou
Wuhan University

In this talk, I will report some constructions of the long-time dynamics for the $SO(4)$-equivariant Yang-Mills heat flow with structure group $SU(2)$ in dimension 4. For a class of initial data with specific decay at spatial infinity, we prove that the solutions constructed can be described by the initial data in a unified manner, generating various blow-up, blow-down, and more exotically, oscillatory solutions. This is a joint work with Y. Sire (JHU), J. Wei (CUHK), and Y. Zheng (TJU).

Blow-up rate for the subcritical semilinear heat equation in non-convex domains

Erbol Zhanpeisov
Tohoku University

We study the blow-up rate for solutions of the subcritical semilinear heat equation. We prove type I estimates for sign-changing solutions in possibly non-convex domains, extending previous results that required convexity or positivity assumptions. The proof uses the Giga-Kohn energy together with a geometric inequality controlling the effect of non-convexity. This is based on joint work with Hideyuki Miura and Jin Takahashi.

Asymptotic behavior of solutions to the Fujita equation in 5D and 6D

Junichi Harada
Akita University

We construct new solutions to the Sobolev critical Fujita equation in 5D. A new feature of these solutions is that the center of mass escapes to spatial infinity as t to infinity. Furthermore, we present a bubble tower phenomenon in 6D.

Harmonic maps into singular spaces

Yannick Sire
Johns Hopkins University

The heat flow of harmonic maps from a smooth, compact Riemannian manifold without boundary, (M, g) into another smooth, compact Riemannian manifold without boundary (N, h) was first studied in the seminal work of Eells and Sampson when the target manifold (N, h) has non-positive curvature. R.Hamilton studied the case when M has a compact smooth boundary (and some special cases when N has also a smooth, compact boundary). Gromov-Schoen studied harmonic maps from M into a singular $\mathrm{Cat}(0)$ space which was used to understand the p-adic superrigidity of lattices in groups of rank one. A key analytical property of such harmonic maps is the Lipschitz continuity, from which one derives Bochner type estimates and vanishing theorems. As for Eells-Sampson theorem, it is rather natural to study the associated gradient (heat) flow, and it has been a long open problem to construct suitable weak solutions in the singular setting. In this talk, I shall describe an elliptic approach (which goes back to De Giorgi and also T. Ilmanen in the 1990s) to this problem both in the smooth and the singular settings, i.e. when the target is $\mathrm{CAT}(0)$ space. I will explain how to get Lipschitz bounds in the space variables (hence a suitable solution of the flow) and how this new approach offers as well a new viewpoint on the old problem of mappings between smooth manifolds. This is joint work with FH Lin, A Segatti and C Wang.