Workshop on Nonlinear parabolic PDEs and related fields
- in honor of the 60th birthday of Marek Fila and Peter Poláčik -

Abstract

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at The University of Tokyo

Lecture Room, Graduate School of Mathematical Sciences

How rates affect rates - some treasure discoveries related to the work of Marek Fila

Michael Winkler (Paderborn University)

Some important developments in the work of Marek Fila address questions concerning the influence that spatial asymptotics of initial data may exert on solution behavior in Cauchy problems for various parabolic problems. The purpose of this presentation consists in reviewing selected findings in such directions, with a special focus on some particularly subtle results that seem hardly accessible to classical functional analytical approaches.

Convergence to traveling wave for the logarithmic diffusion equation with bistable nonlinearity on a line

Masahiko Shimojo (Okayama University of Science)

The logarithmic diffusion equation $\partial_t w = \Delta(\log w)$ has been studied in various contexts. It appears as the expansion into a vacuum of a thermalized electron cloud, and the central limit approximation of Carleman's model of the Boltzman equation. Another fantastic relation can be found with the Ricci flow on a plane.

In the introductory part of this talk, some results on extinction phenomenon of the Cauchy problem of $\partial_t u = \partial_x^2(\log u)$ will be explained. The main part of this talk will be the investigation on the behavior of solutions to the following logarithmic diffusion equation:

$$\begin{cases} \partial_t u = \partial_x^2(\log u) + R(u), & x \in \mathbb{R},\ t > 0, \\ \lim\limits_{x\to-\infty}(\log u) = 1, \quad \lim\limits_{x\to+\infty}\partial_x(\log u) = -\beta, & t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}, \end{cases}$$

where $\beta > 0$ is a given constant and $R(u) = u(1-u)$ or $R(u) = u(1-u)(u-a)$ with $0 < a < 1$. Our result claims that the solution approaches a traveling wave as $t \to \infty$. This is a joint work with E. Yanagida, P. Takac, H. Matsuzawa and H. Monobe.

Entire solutions of a semilinear heat equation

Pavol Quittner (Comenius University)

We examine entire solutions of the semilinear heat equation $u_t = \Delta u + u^p$ on R^N, where p is supercritical in the Sobolev sense. We prove a new Liouville-type theorem saying that if p is greater than the Lepin exponent, then all positive bounded radial entire solutions are steady states. The theorem is not valid without the assumption of radial symmetry; in other ranges of supercritical p it is known not to be valid even in the class of radial solutions. Our other results include classification theorems for nonstationary entire solutions (when they exist), as well as some applications in the theory of blowup of solutions. This is a joint work with Peter Poláčik.

Critical Fujita exponents for semilinear heat equations with quadratically decaying potential

Tatsuki Kawakami (Ryukoku University)

We study the existence/nonexistence of global-in-time positive solutions of the Cauchy problem of semilinear heat equations with potential V. It is well known that the critical Fujita exponent, which separate that the problem possesses global-in-time solutions or not, depends on the behavior of the potential V. In particular, the case where V decays quadratically at the space infinity is on the borderline where the critical Fujita exponent can vary in $(1, \infty]$, and there are several partial results. In this talk we identify the critical Fujita exponent for the case where V is a radially symmetric potential decaying quadratically. The identification of the critical Fujita exponent for this problem is a delicate issue, in particular, when the Schrödinger operator $L_V := -\Delta + V$ on L^2 is critical. Indeed, in the critical case, the critical Fujita exponent is different from previous results and it depends on a new threshold number. This talk is based on a joint work with Kazuhiro Ishige (University of Tokyo).

Solvability for a semilinear heat equation without the self-similar structure

Yohei Fujishima (Shizuoka University)

We consider a semilinear heat equation without the self-similar structure. By focusing on some quasi-scaling property and its invariant integral, we develop a classification theory for the existence and nonexistence of local in time solutions, and then we discuss the existence of global in time solutions for small initial data. We also study the nonexistence of global in time solutions for nonnegative initial data. These results gives a generalization of the Fujita exponent for a semilinear heat equation with general nonlinearity. This talk is based on a joint work with N. Ioku (Ehime University).

Propagation of bistable fronts through a perforated wall

Hiroshi Matano (Meiji University)

We consider a bistable reaction-diffusion equation on $\mathbf{R}^N$ in the presence of an obstacle K, which is a wall of infinite span with periodically arrayed holes. More precisely, K is a closed subset of $\mathbf{R}^N$ with smooth boundary such that its projection onto the x_1-axis is bounded, while it is periodic in the rest of variables $(x_2, \ldots, x_N)$. We assume that $\mathbf{R}^N \setminus K$ is connected. Our goal is to study what happens when a planar traveling front coming from $x_1 = +\infty$ meets the wall K.

We first show that there is clear dichotomy between 'propagation' and 'blocking'. In other words, the traveling front either completely penetrates through the wall or is totally blocked, and that there is no intermediate behavior. This dichotomy result will be proved by what we call a De Giorgi type lemma for an elliptic equation on $\mathbf{R}^N$. Then we will discuss sufficient conditions for blocking, and those for propagation. This is joint work with Henri Berestycki and François Hamel.

On the multiplicity of self-similar solutions of the semilinear heat equation

Peter Poláčik (University of Minnesota)

In studies of superlinear parabolic equations

$$u_t = \Delta u + u^p, \quad x \in \mathbb{R}^N,\ t > 0,$$

where $p > 1$, backward self-similar solutions play an important role. These are solutions of the form $u(x,t) = (T-t)^{-1/(p-1)}w(y)$, where $y := x/\sqrt{T-t}$, T is a constant, and w is a solution of the equation $\Delta w - y\cdot\nabla w/2 - w/(p-1) + w^p = 0$. We consider (classical) positive radial solutions w of this equation. Denoting by p_S, p_{JL}, p_L the Sobolev, Joseph-Lundgren, and Lepin exponents, respectively, our main result states for $p \in (p_S, p_{JL})$ there are only countably many solutions, and for $p \in (p_{JL}, p_L)$ there are only finitely many solutions. We will discuss main ideas of the proof and show some applications of the theorem. This is joint work with Pavol Quittner.

Continuation beyond interior gradient blow-up in a semilinear parabolic equation

Marek Fila (Comenius University)

It is known that there is a class of semilinear parabolic equations for which interior gradient blow-up (in finite time) occurs for some solutions. We construct a continuation of such solutions after gradient blow-up. This continuation is global in time and we give an example when it never becomes a classical solution again. This is a joint work with Johannes Lankeit.