

複素解析 II 演習 第5回

問題 5.1.

$$(1) \operatorname{Res}(f, 0) = \frac{1}{2} \qquad (2) \operatorname{Res}(f, 0) = \frac{1}{6}$$

問題 5.2. 積分路は省略.

$$(1) \operatorname{Res}(f, 0) = \frac{1}{6}, \quad \int_{|z|=3} \frac{\sinh z}{z^4} dz = \frac{\pi}{3}i$$

$$(2) \operatorname{Res}(f, 0) = \frac{1}{6}, \quad \int_{|z-1|=2} z^2 e^{1/z} dz = \frac{\pi}{3}i$$

$$(3) \operatorname{Res}(f, 0) = \frac{1}{24}, \quad \int_{|z+2i|=3} z^3 \cosh \frac{1}{z} dz = \frac{\pi}{12}i$$

$$(4) \operatorname{Res}(f, -i) = \frac{e^{-i}}{2i}, \quad \operatorname{Res}(f, -3i) = \frac{e^{-3i}}{-2i},$$

$$\int_{|z+2i|=2} \frac{e^z}{z^2 + 4iz - 3} dz = \pi(e^{-i} - e^{-3i})$$

$$(5) \operatorname{Res}(f, 0) = \frac{1}{3}, \quad \operatorname{Res}(f, 3) = \frac{1}{3(3-i)^2}, \quad \operatorname{Res}(f, i) = \frac{3-2i}{(1+3i)^2},$$

$$\int_{|z-i|=2} \frac{1}{(z^2 - 3z)(z-i)^2} dz = 2\pi i \left(\frac{1}{3} + \frac{3-2i}{(1+3i)^2} \right)$$

$$(6) \operatorname{Res}(f, 0) = \frac{1}{8}, \quad \operatorname{Res}(f, -2) = -\frac{5}{8}e^{-2},$$

$$\int_{|z+2|=4} \frac{e^z}{z(z+2)^3} dz = 2\pi i \left(\frac{1}{8} - \frac{5}{8}e^{-2} \right)$$